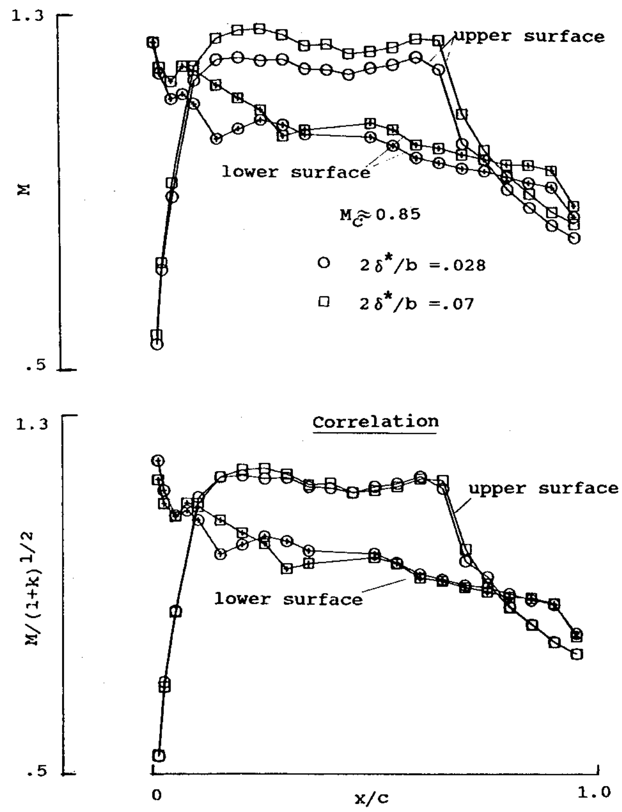

Fig. 2 Correction of local Mach number measurements on a supercritical airfoil at zero incidence with different sidewall boundary-layer thicknesses.[9]

Acknowledgment

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Comparison of Five Methods for Determination of the Wall Shear Stress

Lars R. Saetran*

University of Trondheim NTH, Trondheim, Norway

Introduction

THE wall shear stress τ_w is a crucial parameter for determining transport of mass and energy in ducts and the neighborhood of fixed walls and for determining the drag on constructions immersed in fluid flow. Also, since most of the universal scaling laws for turbulent boundary layers involve the friction velocity, $v_* = (\tau_w/\rho)^{1/2}$, it is of primary importance to have accurate methods for measuring τ_w. The review by Winter[1] explores most of the methods available for direct and indirect measurement of τ_w in turbulent boundary layers. It is considered to be difficult, time consuming, and costly to accomplish accurate direct measurements of the local wall shear stress by using methods involving, for example, floating elements. In the present work, five methods are used to determine τ_w, expressed through the dimensionless skin-friction coefficient c_f, as

$$c_f = 2\tau_w/(\rho U_\infty^2) = 2(v_*/U_\infty)^2 \tag{1}$$

The methods are of the indirect type and are based on the shape of the mean velocity profile.

In order to isolate the variables as far as possible a two-dimensional zero pressure gradient turbulent boundary-layer (TBL) flow with well-defined boundary conditions was established as the test case for comparison of the methods for determination of τ_w. Effort was made in order to achieve what Coles[2] classifies as a "normal" two-dimensional TBL as the test case. A two-dimensional TBL may easily be distorted by factors as three-dimensional flow effects, a poor choice of the tripping device and high values of the freestream turbulence intensity. The effect of a high value of the freestream turbulence level is to affect the mean velocity profile in the outer part of the boundary layer. Blair[3] found that the mean velocity profile is not influenced by freestream turbulence when the freestream turbulence level is less than about 1%. A tripping strip is often used to promote a stable transition to a TBL. A poor choice of this device may cause disturbances from which the TBL may be very slow to recover.[4,5] Three-dimensional effects such as small mean crossflow components may seriously influence the local skin friction when compared to two-dimensional theory.[6,7] Such small mean crossflow components are often hardly measurable. There are, however, several parameters characterizing the development of a normal two-dimensional TBL, which will be reported in the following sections.

Experimental System

The turbulent boundary layer investigated develops on the flat floor in the 500×1000 mm test section of a closed-return wind tunnel where the velocity range is 1–40 m/s and the

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freestream turbulence level is less than 0.4%. The settling chamber is equipped with a honeycomb and screens in order to smooth the flow by breaking up large eddies. A heat exchanger enables control of the freestream temperature within ±0.05°C. The test section roof is slightly increased in the streamwise direction to compensate for boundary-layer growth. Transition to a stable TBL is triggered at the test section entrance by means of a tripping strip 25 mm wide and 1.5 mm high. The probe traverse is mounted independently of the wind tunnel with the probe holder submerged into the airflow through slots in the test section roof. A Mitutoyo No. 192-106 mechanism having divisions of 0.01 mm and a maximum travel of 300 mm is used for the traversing. Velocity measurements are made with single DISA 55P05 hot wires (5 mm in diameter) operated by a DISA 55M01 constant-temperature anemometer at an overheat ratio of 0.8. The signals are linearized in a DISA 55M25 linearizer.

The distance from the wire to the test wall is determined in the following way[8]:

1) The diameter of the probe tips was measured and the position of the wire on the tip found.

2) The probe was installed in the test section and the wire axis was accurately aligned with the test plate by using a theodolite and the mirror image of the probe tips in the polished aluminium test plate.

3) The probe tips were brought in contact with the electrically conducting test plate while the resistance between the probe and the plate was monitored.

4) The probe was traversed away from the wall in very small steps. The point at which the probe tips left the wall, i.e., where the resistance made a jump from zero to infinity, was found. This point is determined with an accuracy of ±0.01 mm. The advantage of this method is that the position of the probe relative to the wall is found while the tunnel is in operation.

Test Case

The longitudinal pressure gradient $\partial\bar{p}/\partial x$ is removed by adjusting the longitudinal inclination of the test section roof, such that the resulting pressure distribution in terms of $2(p-p_{\text{ref}})/\rho U_{\text{ref}}^2$ is constant within ±0.8%. The longitudinal freestream traverses did not reveal any variation (within ±0.3%) of the freestream velocity in the first 4 m of the test section when measured with the hot wire.

Freestream velocity settings used are 10 and 16 m/s. Mean velocity profiles for the first setting are shown in Fig. 1 in the usual dimensionless form u^+ $(=U/v_*)$ and y^+ $(=yv_*/\nu)$. It is well known (see, e.g., Ref. 9) that the TBL may be divided in a wall region and a wake region. The wall region is considered as unaffected by the history of the boundary layer, pressure gradient, and other manipulations of the outer conditions. Spalding's[10] law of the wall presents a single analytically smooth expression for the whole wall region as

$$y^+ = u^+ + A\left[\exp(\kappa u^+) - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2} - \frac{(\kappa u^+)^3}{6} - \frac{(\kappa u^+)^4}{24}\right] \tag{2}$$

where the constants are $A=0.015$ and $\kappa=0.53227$. This particular choice of the constants was suggested by Persen[11] to fit the mean velocity profile in the wall region only and is different from that of Spalding[10] who found a best fit throughout the whole boundary layer with $A=0.1108$ and $\kappa=0.4$. Equation (2) is shown in Fig. 1 in which the following Logarithmic law of the wall is also plotted:

$$u^+ = 2.44\ln(y^+) + 5.4 \tag{3}$$

The displacement thickness δ_1 and the momentum loss thickness δ_2 are defined as

$$\delta_1 = \int_0^\infty \left(1-\frac{U}{U_\infty}\right)\mathrm{d}y, \qquad \delta_2 = \int_0^\infty \left(1-\frac{U}{U_\infty}\right)\frac{U}{U_\infty}\,\mathrm{d}y \tag{4}$$

where U_∞ is the mean freestream velocity and δ the boundary-layer thickness defined as the distance from the wall to the point where the mean velocity has reached 99% of the freestream velocity. The shape factor $H=\delta_1/\delta_2$ characterizes the mean velocity distribution, i.e, the shape of the mean velocity profile. It is plotted in Fig. 2 as a function of $Re_{\delta 2}$ $(=U_\infty\delta_2/\nu)$. The parameters δ_1 and δ_2 are found by integrating the measured mean velocity profile using the trapezoid method. The number of data points is sufficiently large for the method to be accurate. Coles' relation[2] for the shape factor H is also plotted in Fig. 2 and it can be seen that the agreement with the present data is very good. The results from the experiments of Purtell et al.[12] are shown for comparison.

Maximum deviation of mean velocity in the outer layer (wake region) from the logarithmic distribution [Eq. (3)] is given by Δu^+. This deviation characterizes the strength of the wake and is shown in Fig. 2. The present values of Δu^+ are in reasonable agreement with Coles' proposed asymptotic function.[2] An anomalous mean velocity distribution in the wake region, when compared to that of a "normal" two-dimensional TBL, is expected to be reflected in the "wake strength" parameter Δu^+ and may be caused by historical effects from the tripping device or by a high freestream turbulence level. Coles[2] estimated the uncertainty in finding Δu^+ from the experimental data to be 5–10%. In the present work, the maximum difference from Coles' proposed asymptotic function is 5.6%. The agreement is satisfactory and thus indicates that the present TBL behaves "normally."

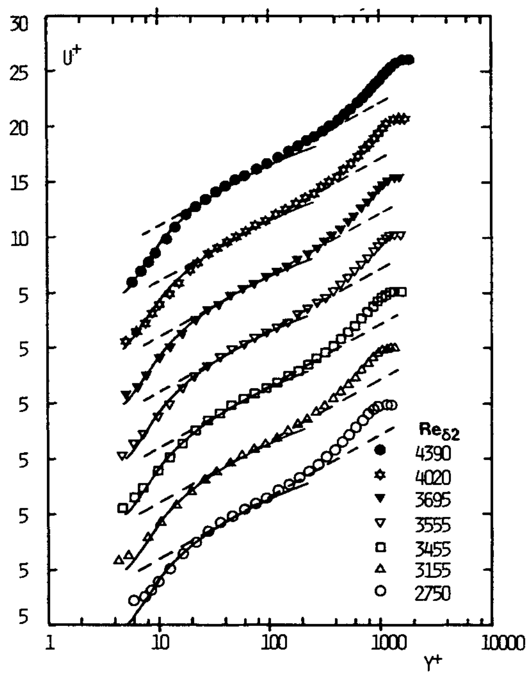

Fig. 1 Profiles of mean velocity [ordinate scale refers to highest curve, other curves successively displaced downwards by 5 units; solid line plots Eq. (2), broken line Eq. (3)].

A test for that a zero pressure gradient two-dimensional TBL is achieved, and also a test for that the data are not influenced by probe interference can be made by checking the experimental data against the von Kármán momentum/integral relation (see, e.g., Ref. 13) for zero pressure gradient two-dimensional flow

$$\frac{\mathrm{d}\delta_2}{\mathrm{d}x}=\left(\frac{v_*}{U_\infty}\right)^2 \tag{5}$$

Anomalies in the flow configuration or in the way the data were taken should then turn out as inequality between the two terms in Eq. (5). Although there is a large inherent uncertainty in the calculation of $\mathrm{d}\delta_2/\mathrm{d}x$, the method is used as a test of the data. The v_* values are found from Eq. (2). The results are presented in Fig. 2 and the ratio of the two terms is (except for at the first point) less than 5%, a result which clearly makes the present TBL (in Coles' terms[2]) to be classified as "normal."

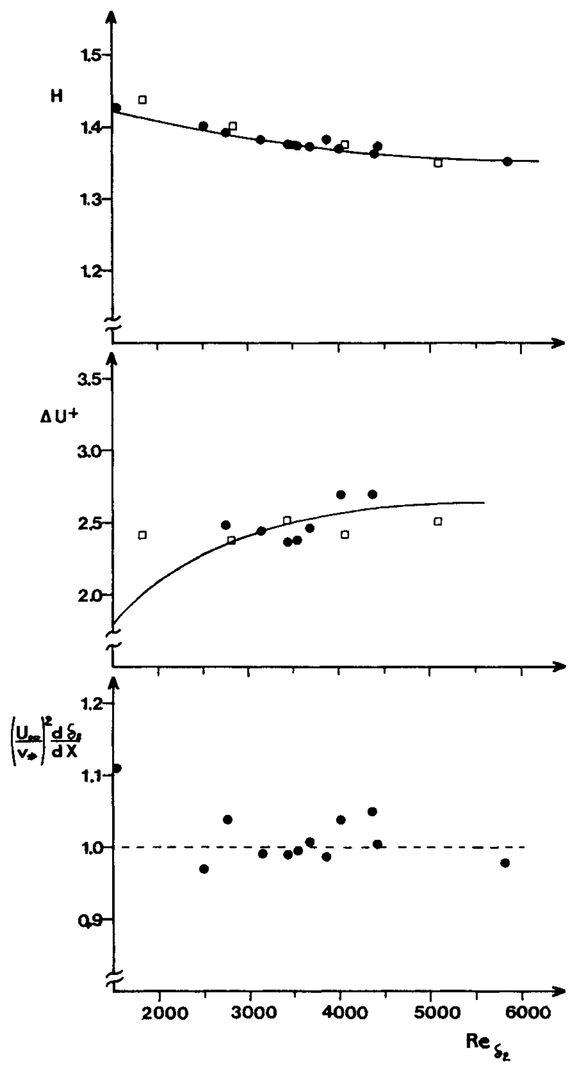

Fig. 2 Shape factor, wake strength parameter, and balance of the momemtum/integral equation as functions of Reynolds number (open symbols represent Purtell et al. data, solid lines Coles' functions).

Results and Discussion

The first method for finding the wall shear stress utilizes Eq. (2). Since the mean velocity U and the distance from the wall y are measured quantities and since both nondimensional quantities u^+ and y^+ are defined through the shear velocity v_*, a simple iteration procedure determines the value of v_* (and thereby the local wall shear stress τ_w) for which the measured values will satisfy Eq. (2).

The second method implies the assumption that the mean velocity profile is linear very near the wall (the viscous sublayer). The wall shear stress is determined by measuring the mean velocity at a point in this region and the distance from the wall to this point. For the present measurements, the position of this point is in the range $5<y^+<8$. Since the assumption of a linear mean velocity profile may be questionable for y^+ values as large as $y^+=8$ and considering the uncertainty in the measured distance from the wall (estimated to ± 0.01 mm), this method is less reliable than the first method. The von Kármán momentum/integral relation in the form

$$c_f=\frac{2\mathrm{d}\delta_2}{\mathrm{d}x} \tag{6}$$

is used as the third method. The uncertainty in the determination of $\mathrm{d}\delta_2/\mathrm{d}x$ is mentioned above. An exponential best-fit function is calculated from the δ_2 values, and the values for $\mathrm{d}\delta_2/\mathrm{d}x$ are found by derivation of this function. However, although the skin-friction coefficient may not be determined by the highest degree of accuracy by this method, it serves as a good test for anomalies in the flow configuration. The fourth method utilizes the relation given by Clauser[6]

$$c_f=2[(1-1/H)/G]^2 \tag{7}$$

where the "defect shape parameter" G is defined as

$$G=\int_0^\delta\left(\frac{U_\infty-U}{v_*}\right)^2\mathrm{d}y\Big/\int_0^\delta\left(\frac{U_\infty-U}{v_*}\right)\mathrm{d}y \tag{8}$$

The parameters H and G are found by integrating the mean velocity profile throughout the boundary layer. The fifth method used to determine the skin-friction coefficient is the empirical relation proposed by Ludwieg and Tillmann[14]

$$c_f=0.246\times 10^{-0.678H}(Re_{\delta_2})^{-0.268} \tag{9}$$

which is supposed to be valid in the range $10^3<\mathrm{Re}_{\delta_2}<10^4$. The experimental data for c_f, determined by the five methods, are presented in Fig. 3 along with the curve of Coles[2] and the experimental data of Purtell et al.[12] The maximum difference between the results at any station is 7%, a result that is satisfactory considering the uncertainty involved in finding c_f from the various methods.

Conclusions

Five methods are used for determining the local wall shear stress in a TBL. Effort is made to check for any anomalies in the flow in order to verify a test case suitable for comparison with two-dimensional theory. The five methods compare well with a maximum of 7% deviation from each other.

When Spalding's law of the wall is used to fit the mean velocity data in the wall region, it is a useful and simple tool for finding τ_w. Due to the good fit in the whole wall region, it compares favorably in practical use with the Logarithmic law of the wall because more data points can be used to determine τ_w. Direct measurements of the linear mean velocity gradient in the viscous sublayer may be used for finding τ_w, but is less reliable because substantial accuracy is required in measuring the distance between the probe and the wall. The use of the von Kármán momentum/integral relation requires a significant number of velocity profiles to be measured and integrated in order to calculate the streamwise

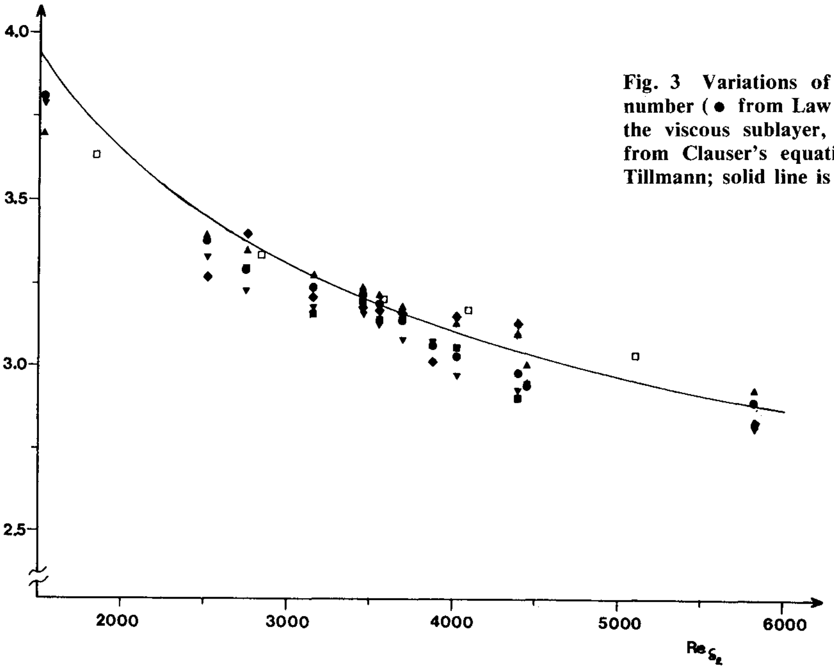

Fig. 3 Variations of the skin-friction coefficient with Reynolds number (● from Law of the wall, ■ from mean velocity profile in the viscous sublayer, ◆ from momentum/integral equations, ▼ from Clauser's equation, ▲ from the relation of Ludwieg and Tillmann; solid line is Coles' distribution).

gradient of the momentum-loss thickness with reasonable accuracy. Both Clauser's relation and Ludwieg and Tillmann's relation for c_f are simple to use because no gradients of parameters entering the equations are required. They do require integral parameters be calculated and will, therefore, be sensitive to any flow anomalies in the outer part of the boundary layer. The overall conclusion from the present study is that Spalding's law of the wall (when applied to the wall region only) is to be preferred for the determination of the local wall shear stress.

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Wind-Tunnel Wall Corrections on a Two-Dimensional Plate by Conformal Mapping

Heinz Däppen*
Eidgenössische Technische Hochschule
Zürich, Switzerland

I. Introduction

THE problem of wall interference is of practical interest, because in aerodynamics it is not always possible to test a model in unconstrained freestream flow. This problem is dealt with in Ref. 1. One approximate formula given there treats the case in which the profile is a plate in a two-dimensional, steady, and irrotational ideal flow (i.e., inviscid and incompressible). The general drawback of such formulas is their inaccuracy when the airfoil has relatively large chord c.

This Note introduces a conformal mapping method for computing this ideal flow and the resulting lift exactly. In our method, the domain between the profile and the tunnel walls is mapped conformally onto an annulus using a Schwarz-Christoffel map for doubly connected regions (see Ref. 2, Sec. 17.5). In order to compute this map numerically, we have to

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*Research Assistant.